ELECTROSTATICS

TOPIC 1

Introduction:

All of us had read about charge in class VII and VIII. A few examples are followings-

- When we take off woolen sweater a cracking sound with light can be observed.
- When we rub plastic scale on our hairs it becomes capable to attract paper pieces.
- Lightning occurs due to accumulation of charge.

Discovery:

Historically the credit of discovery of the fact that amber rubbed with wool or silk cloth attracts light objects goes to Thales of Miletus, Greece, around 600 BC.

- The name electricity is coined from the Greek word elektron meaning amber.
- The experiments on pith balls suggested that there are two kinds of electrification and we find that (i) *like charges repel* and (ii) *unlike charges attract* each other.
- The property which differentiates the two kinds of charges is called the *polarity* of charge.
- The charges were named as *positive* and *negative* by the American scientist Benjamin Franklin.
- By convention, the charge on glass rod or cat's fur is called positive and that on plastic rod or silk is termed negative. If an object possesses an electric charge, it is said to be electrified or charged. When it has no charge it is said to be neutral.

Properties: 1. Additivity of charges : If there are more than a charge then total charge is the sum

of these charges with their sign.

2.Conservation of charges : Charges can neither with created nor be destroyed but can be transferred from one object to another. OR

The total charge of the isolated system is always conserved.

3. Quantization of charges : all free charges are integral multiples of a basic unit of charge denoted by e. Thus charge q on a body is always given by

q = ne

Unit : The unit of charge is coulomb. Prefixes and suffixes can be used to change unit according to the magnitude of charg

Conductors: Those which allow electricity to pass through them easily are called *conductors*.

Insulators: Those which do not allow electricity to pass through them easily are called *Insulators.*

Earthing: When we bring a charged body in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through the connecting conductor (such as our body). This process of sharing the charges with the earth is called *grounding or earthing*.

Methods of charging: 1.By Induction 2.By Friction 3. By touching a charged body

- Questions: 1. Who discovered charge?
 2. If an object is charged successively by +5C, -7C and +3C charge. What is the total charge on that object?
 3. The rate of flow of charge if called as electric current (I = Q/t). If 9.5 electrons are moving through a conductor in 5 seconds and one electron carries 1.6 x 10⁻¹⁹C charge then find the current flowing through the conductor.
 4. How many electrons are needed to form one coulomb of charge.
- Note: 1. When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor. In contrast, if some charge is put on an insulator, it stays at the same place.

2. Electrostatics is made of electro (means charge) and statics (means at rest) a branch to study the properties of charges at rest.

Topic-2 / Electrostatics / Coulomb's law (Force between charges at rest)

Coulomb's law is a quantitative statement about the force between two point charges.

Point Charges : When the linear size of charged bodies are much smaller than the distance separating them, the size may be ignored and the charged bodies are treated as *point charges*.

Statement: The electrostatic force acting between two point charges is -

- (a) Directly proportional to the product of charges and
- (b) Inversely proportional to the square of their distance.
- (c) It acts along the line joining the charges.

i.e. If two charges q and Q are placed in air at a distance r from each other then $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$

Q °----⊃

Where ε_0 is called as electric permittivity of free space / vacuum and $\varepsilon_0 = 8.854 \times 10^{-12} C^2 / (N-m^2)$

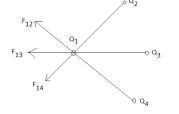
Limitations of coulomb's law:

- 1. The charges should be point charges.
- 2. The charges must be at rest.
- 3. The charges must not radiate energy.

COULOMB'S FORCE DUE TO MULTIPLE CHARGES / SUPERPOSITION PRINCIPLE

The net force acting on a charge due to multiple charges is the vector sum of all the forces acting on the given charge-

Suppose there are four charges Q_1 , Q_2 , Q_3 and Q_4 as shown.



Then net force on charge Q_1 due to Q_2 , Q_3 and Q_4 will be the sum forces between $(Q_1 \& Q_2)$, $(Q_1 \& Q_3) \& (Q_1 \& Q_4)$ i.e. $F_1 = F_{12} + F_{13} + F_{14}$

Questions:	 State Coulomb's law of electrostatics. Write limitations of coulomb's law of electrostatics. Two charges ± 2 C are placed along a horizontal line at a distance of 2 cm 		
from			
	each other. Find the net electrostatic force on a charge of 2 C placed at a point P along the same line at a distance of 3 m from -2 C charge, in air.		
	$\begin{array}{cccc} +2C & & 2C \\ & & & & \\ A & & B & & P \\ < & & & 2m & & & 3m & \end{array}$		
	4 . In question 3, if point P is lying on perpendicular bisector of AB and above it at a perpendicular distance of 2 m and 2 C charge is placed at P then find the net force on the charge at P.		
	5. Define dielectric constant in terms of Coulomb's force.		
Note:	1. The value of $\frac{1}{4\pi\epsilon_0}$ is 9 x 10 ⁹ N-m ² /C ²		
	2. If two charges q and Q are placed in a medium whose dielectric constant is K then the electrostatic force between them becomes		
	$ec{f F}=rac{1}{4\pi K\epsilon_0}rac{qQ}{r^2}\hat{r}$, where r is the distance between the charges		

Topic-3 / Electrostatics /

Permittivity and Polarization

Permittivity is analogous to the conductivity of a conductor, similar to the conductivity which is the property of a material by which it allows free flow of electrons through it, Permittivity is the property of a material by which it allows the free flow of electric field through it.

Polarization: If we place two opposite charges at a distance then electric field is formed in between the space directed from the positive charges to the negative as shown

Now if we place a substance or another medium in between those plates, Initially all the atoms and charges in the medium are unpolarized are they are scattered.

But because of the force exerted on the charges in the medium by the electric field, the charges and atoms in the medium are polarized or they all are aligned in the same direction.

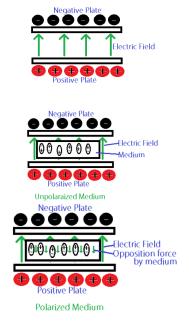
When the medium is polarized, an opposing electric field is developed in the medium.

"The property of a material by which despite the presence of opposing electric field , electric field is allowed to pass through the material is called it's Permittivity."

Relative Permittivity: Relative Permittivity (also termed as 'Dielectric constant K or ϵ_r) of a medium is the ratio of its Permittivity to the Permittivity of free space. i.e. $\epsilon_r = \frac{\epsilon_m}{\epsilon_r}$

Questions:

- 1. What is electric polarization?
- 2. What is the unit of dielectric constant?
- 3. Define dielectric constant in terms of-
- (a) Capacity of a parallel plate capacitor.
- (b) Electrostatic force.
- 4. What is the minimum value of dielectric constant and for which material?
- 5. What is the value of dielectric constant for metals.



Topic-4 / Electrostatics

Electric field (\vec{E})

Every charge produce its own 'Electric field'. The term field means region. A region around a charge in which any other charge experiences electrostatic force.

The strength of electric field is termed as electric field intensity.

High magnitude of electric field intensity implies heavy electrostatic force and vice versa. Electric field intensity due to a point charge 'Q' at a distance 'r' from it is given by-

 $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$, Where $\hat{r} = \frac{\vec{r}}{|r|}$

Electric field due to a **positive charge** '+Q' is **represented radially outward** in nature in all possible direction in three dimensional space, whose origin can be considered at the position of positive charge.

While Electric field due to a **negative charge '+Q' is represented radially inward** in nature from all possible direction in three dimensional space, whose origin can be considered at the position of positive charge.

Mathematically,

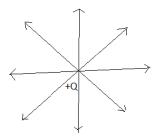
Electric field is defined as electrostatic force acting on a unit positive charge.

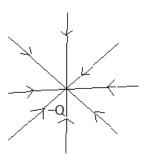
i.e.

$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$

$$\vec{E} \equiv \frac{\vec{F} = \frac{1 \quad Qq_0}{\vec{F} = \frac{1 \quad Qq_0}{4\pi\epsilon_0 \ r^2} \hat{r}}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$





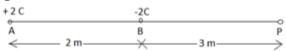


Electric field due to multiple charges (Principle of superposition) * similar to coulomb's force The net electric field at a point due to multiple charges is the vector sum of electric fields due to each charge at that point-Suppose there are three charges Q₁, Q₂, and Q₃ as shown. Then the net electric field at poit P due to these charges can be found as- $\overrightarrow{E_{NET}} = \overrightarrow{E_1} + \overrightarrow{E_2} + \overrightarrow{E_3}$

Questions:

1. What is the unit and dimensional formula of electric field intensity.

2. Two charges $\pm 2C$ are placed along a horizontal line at a distance of 2 cm from each other. Find the net electric field at a point P along the same line at a distance of 3m from -2 C charge, in air.



3. In question 2, if point P is lying on perpendicular bisector of AB and above it at a perpendicular distance of 2 m. Find the net electric field at point P.

Topic-5 ELECTRIC FIELD LINES AND ELECTRIC FLUX

Electric Field Lines: (Properties)

1. These are imaginary lines around any charge.

2. They are outward for positive charge.

3. They are inward towards a negative charge.

4. Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.

5. In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.

6. The tangent to them at any place gives the direction of electric field at that place.

7. They cannot intersect each other as at the point of intersection two tangents are

possible and hence two directions of electric field, which is impossible.

8. Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field

Electric field lines representation:

Uniform field (Parallel lines)	Pair of opposite charge	Pair of similar charges
	+q +q	

Electric flux (ϕ **) :** It is the measure of electric field lines passing through unit area placed normally to electric field lines.

 $\varphi = \int \vec{E} \cdot \vec{dS}$ Its unit is N-m².C⁻¹.

Questions:

- 1. Two Electric field lines can never intersect each other. Justify.
- 2. "Electric field is conservative." Which property of field lines supports this statement.
- 3. Draw electric field lines for-
 - (a) A pair of similar positive charges,
 - (b) A pair of similar positive charges and
 - (c) A pair of equal and opposite charges

Electric dipole (Di means two)

It is a system of two charges of equal magnitude but opposite polarity.In other words, Electric dipole is a system of two equal and opposite charges separated by a very small distance.**e.g**. In hydrogen atom, 1 orbital electron having charge (-e) and a proton in the nucleus having a charge(+e) constitute an electric dipole.

Electric dipole moment (\vec{p}) :

It is the strength of any dipole. It is defined as the product of magnitude of either charge and the distance between the charges. If two charges +q and -q are separated by a distance '2I' then Electric dipole moment of this dipole

> $\vec{p} = q \times 2l$ (Directed from -q to +q) Electric field of an electric dipole:

Case-I (For points on axis):

Electric field at a point P due to positive charge(+Q)- $\vec{E_1} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{AP^2} \cdot \hat{p}$ from A to P $\vec{R_1} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{AP^2} \cdot \hat{p}$ from A to P

and Electric field at a point P due to negative charge(-Q)-

$$\overrightarrow{E_2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{BP^2} \cdot \hat{p}$$
 from P to A

Where \hat{p} is a unit vector along -Q to +Q.

Therefore according to principle of superposition-

Net electric field at P

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{BP^2} - \frac{1}{AP^2} \right] \hat{p}$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{(r-L)^2} - \frac{1}{(r+L)^2} \right] \hat{p}$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0} \left[\frac{(r+L)^2 - (r-L)^2}{(r-L)^2 \cdot (r+L)^2} \right] \hat{p}$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{4rL}{(r^2 - L^2)^2} \hat{p}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \hat{p} \quad For \ r \gg L.....Result$$

Topic-7 / Electrostatics

Electric dipole (continued from-1)

Case-II (For points on equatorial plane):

Electric field at a point P due to positive charge(+Q)-

$$\overrightarrow{E_1} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{AP^2}$$
 from A to P

and Electric field at a point P due to negative charge(-Q)-

 $\overrightarrow{E_2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{BP^2}$ from P to B

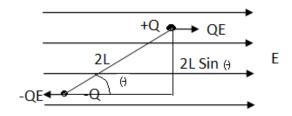
(use law of parallelogram to find their resultant magnitude and direction)

 $\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \hat{p} \qquad For \ r \gg L.....Result$

Electric dipole placed in a Uniform Electric Field:

When an electric dipole is placed in a uniform electric field, both the charges experiences equal and opposite force $(\pm QE)$ and hence the

Net Force acting on electric dipole is zero.



But,

Both the forces are having different lines of action and hence a torque will act on it. Which is given by-

 $\ensuremath{\mathbb{T}}$ = Either force x Perpendicular distance between them

 \Rightarrow T = QE x 2L Sin Θ

- ⇒ Т = pE Sin ө
- \Rightarrow T = $\vec{p} \times \vec{E}$Result

Questions:

1. Define electric dipole moment. Write its SI unit.

2. Whether electric dipole moment is a scalar or vector? If vector then give its direction.

3. What is the net force and torque acting on an electric dipole placed in a uniform electric field.

- 4. Find the net electric field due to an electric dipole at an axial point.
- 5. Find the net electric field due to an electric field at an equatorial point.
- 6. What are polar and non-polar molecules?
- 7. Find the work done to rotate the electric dipole placed in a uniform electric field.

OR

Find the potential energy of an electric dipole.

Note: One can find electric potential at a point on axis of electric dipole as well as at a point on equatorial plane of electric diploe by keeping in mind that electric potential is a scalar quantity.

Continuous Charge Distribution and Gauss's Law

Charge can be uniformly distributed in one /two /three dimensional objects. This distribution of charges on conductors is termed as **charge density**.

Linear Charge Density (Λ): When charge (q) is uniformly distributed over a wire of length L, the charge per unit length is called as linear charge density (Λ). Its unit is coulomb per meter.

i.e.
$$\lambda = \frac{q}{L}$$

<u>Surface Charge Density (σ):</u>

When charge (q) is uniformly distributed over a surface area A, the charge per unit surface area is called as surface charge density (σ). Its unit is coulomb per meter².

i.e.
$$\sigma = \frac{q}{A}$$

Volume Charge Density (p):

When charge (q) is uniformly distributed over a volume V, the charge per unit volume is called as volume charge density (ρ). Its unit is coulomb per meter³.

i.e. $\rho = \frac{q}{V}$

<u>Gauss's Law of electrostatics</u>: Electric flux through a closed surface S is $\frac{1}{\epsilon_0}$ times the total charge enclosed by the surface.

i.e.
$$\phi = \frac{q_{enclosed}}{\epsilon_0}$$

Questions:

1. State gauss's law of electrostatics.

2. Give the directions of electric field for (i) $\lambda > 0$ & (ii) $\lambda < 0$.

3. In the adjoining figure find electric flux through the surface-

(i) S_1 and (ii) S_2

4. The electric field components in Figure are $Ex = a \times 1/2$, Ey = Ez = 0, in which a = 800N/C m^{1/2}.

Calculate

(a) the flux through the cube, and

(b) the charge within the cube. Assume that a = 0.1 m.

5. An electric field is uniform, and in the positive x direction for +x, and uniform with the same magnitude but in the - x direction for negative x. It is given that E = 200 î N/C for x > 0 and E = -200 î N/C for x < 0. A right circular cylinder of length 20 cm and radius 5 cm has its centre at

the origin and its axis along the x-axis so that one face is at x = +10 cm and the other

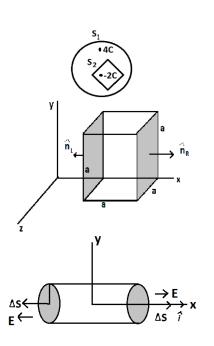
is at x = -10 cm.

(a) What is the net outward flux through each flat face?

(b) What is the flux through the side of the cylinder?

(c) What is the net outward flux through the cylinder?

(d) What is the net charge inside the cylinder?



Topic-9 / Electrostatics Applications of Gauss's Law

Gauss's law of electrostatics can be used to find electric field due to -

- One dimensional charge distribution (Uniformly charged wire)
- Two dimensional charge distribution (Uniformly charged surface)
- Three dimensional charge distribution (Uniformly charged thin spherical shell (field inside and outside))
- A point charge,

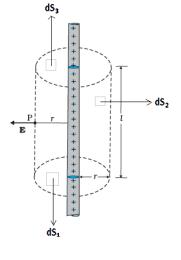
METHOD OF APPLICATION:

- Assume and draw a hypothetical Gaussian surface to enclose -
 - * any part of uniform charge distribution
 - * or total charge given
- Gaussian surface must be symmetrical around charge or charge distribution¹ and must pass through the point² where electric field is to determine.
- Find electric flux through this Gaussian surface by
 - (i) Gauss's law, i.e. $\phi = \frac{q_{enclosed}}{s_{enclosed}}$
 - (ii) Definition of electric flux, i.e. $\phi = \int \vec{E} \cdot \vec{dS}$
- Eliminate ϕ from both and find electric field.

CASE-I: Electric field due to infinitely long straight wire:

Previous knowledge required: Electric field is normal to uniformly charged wire/ sheet.

Step 1: Imagine a cylindrical Gaussian surface of radius r (as the field is required at a distance 'r' from charged wire) and of length 'l'(length depends on observer because in place of charge 'charge distribution' is given). **Step 2:** According to Gauss's law, $\Phi = \frac{q_{enclosed}}{c}$ $\Rightarrow \quad \Phi = \frac{\lambda l}{\epsilon_0}$ (1) Step 3: According to definition of electric flux, i.e. $\phi = \int \vec{E} \cdot \vec{dS}$ but there are three surfaces (Two circular & one cylindrical) $\Rightarrow \quad \phi = \int \vec{E} \cdot \vec{dS}_1 + \int \vec{E} \cdot \vec{dS}_2 + \int \vec{E} \cdot \vec{dS}_3$ ⇒ $\phi =$ $\int E \cdot dS_1 \cdot \cos 90^0 + \int E \cdot dS_2 \cdot \cos 0^0 + \int E \cdot dS_3 \cdot \cos 90^0$ $\Rightarrow \quad \varphi = \mathbf{0} + \int E \cdot dS_2 \cdot \mathbf{1} + \mathbf{0}$ $\Rightarrow \phi = E.2\pi r.l....(2)$



Step 4: From equation (1) and (2)-

 $\frac{\lambda l}{\epsilon_0} = \mathbf{E} \cdot 2\pi \mathbf{r} \cdot \mathbf{l}$ $\Rightarrow \vec{\mathbf{E}} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{\mathbf{r}} - ----(\text{Result})$

Questions: 1. Using Gauss's law, find the electric field due to a uniformly charged infinitely large thin sheet, near it.

> Also draw the graphical variation of electric field with the distance from charged sheet.

2. Using Gauss's law, find the electric field due to a uniformly charged thin shell at a point-

- Inside the cell. (i)
- on the shell and (ii)
- outside the shell. (iii)

Also draw the graphical variation of electric field with the distance from centre of the shell.